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AN HYPOTHESIS ON THE UNIVERSALITY OF EJECTION PROPERTIES OF TURBULENT GAS JETS AND ITS APPLICATION

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Transforming the Navier-Stokes equations for the case of turbulent fluid motion by replacing the actual parameters of the moving medium by a sum of some average value and by a pulsation component leads to an extremely complicated system, the solution which is usually attempted by two methods: with the aid of a statistical ensemble or by using certain semi-empirical relationships. The semi-empirical theory of turbulence allows some results to be obtained, in particular, the solution to the problem of the propagation of a free turbulent jet of incompressible fluid in a quiescent medium. However, this problem had remained unsolved until recently because of the absence of the appropriate semi-empirical relationships needed to close the system of equations which describe the propagation of a gas jet in a moving medium. And only after the work of G. N. Abramovich [1], who, by developing L. Prandtl's idea, found a semi-empirical equation relating the intensity of the turbulent expansion of the jet to the values of the flow velocity at the boundaries of the mixing zone, were the problem of the propagation of a fluid jet in a flow with the same physical properties and the problem of gas jets in moving and quiescent media [2-4] solved.

A comparison between obtained solutions and the experimental data of different authors [3] has shown that semi-empirical theory properly reflects the general outlines of the actual process of gas jet propagation. The systematic discrepancy observed between the theoretical and experimental results when

$m_o (= u_\infty / u_o) > 0.35$ is qualitatively explained by the effect of the initial turbulence of the intermixing flows, which is not accounted for in the theory. However calculations show that the initial degree of this turbulence should be very high, of the order of 10-15%. Since for steady-state flow in different channels the degree of turbulence does not usually exceed 1-5% (these same values characterize the turbulence of a jet and concomitant flow), it is clear that the abovementioned qualitative explanation is gravely in need of experimental grounding.

Calculation of the acquired flow rate of a gas jet being propagated in a quiescent or moving medium has shown that in the region where the theoretical and experimental characteristics of the jet coincide the ejection properties of the jets remain the same as for a corresponding submerged jet (with the same initial momentum and cross-sectional area of the exhaust nozzle) of incompressible fluid.

This fact suggests the idea of the universality of the ejection properties of a turbulent jet under whatever external conditions its growth occurs. The expressed hypothesis permits us without particular difficulties to use the well known and extensively tested theory of a submerged turbulent jet of incompressible fluid to find the laws for the propagation of a jet under more complicated conditions (nonisothermic state, off-design efflux, dynamic compressibility, longitudinal pressure gradients, etc.).

We note that this hypothesis is based on the following considerations: since the basic characteristic of a jet is its momentum; and its chief property, the ejection of substance from the ambient medium, it is natural to assume that when the momentum of the jet is kept constant, its ejection properties remain unchanged.

Analytically, the hypothesis concerning the universality of the ejection properties of turbulent jets is formulated in the following way. If a given jet propagating under ordinary conditions (subscript 1) and a standard submerged isothermic free jet (subscript a) issue from identical nozzles and have equal momenta at the initial cross section

$$M_1 = M_a \tag{0.1}$$

then these jets also possess the same ejection properties. In other words, the laws for the increase in the mass of a jet due to the substance drawn into it from the ambient medium coincide for the given and standard jets when condition (0.1) is fulfilled:

$$\frac{dG_i}{dx} = \frac{dG_a}{dx} \qquad (0.2)$$

Here G is the gas flow rate in an arbitrary cross section of the jet, and $\underline{x}$ is the longitudinal coordinate referred to the nozzle edge.

It is easy to show that when condition (0.1) is observed and when the areas of the outlet apertures are equal (besides having the same geometric shape) the ratio of the flows of the standard and arbitrary jet is expressed by the formula

$$\frac{G_{a0}}{G_{i0}} = \sqrt{\frac{\rho_{ae}}{\rho_{i0}}} = \sqrt{\rho^{\circ}} \qquad (0.3)$$

where, by assumption, ρ_{ae} is the density of the ambient medium into which the jet issues.

Let us use formula (0.3) to reduce relationship (0.2) to dimensionless form:

$$\frac{dG_i^{\circ}}{dx^{\circ}} = \sqrt{\rho^{\circ}}\,\frac{dG_a^{\circ}}{dx^{\circ}} \qquad \left(G^{\circ} = \frac{G}{G_0},\ x^{\circ} = \frac{x}{r_0}\right) \qquad (0.4)$$

where r_0 is the initial radius of an axially symmetrical jet or the initial half-width of a plane-parallel jet.

The hypothesis proposed above may be formulated in yet another way: the intensity of the accumulation of mass by the jet (the jet's ejection capacity) is proportional to its initial momentum dG/M = idem.

If there are two jets flowing out from geometrically similar nozzles, it is not difficult to show that the ratio of the reduced increments ($dG^{\circ} = dG/G_0$) in these jets may be expressed by the formula

$$\frac{dG_1'}{dG_2'} = \sqrt{\frac{M_1}{M_2}\frac{F_2}{F_1}\frac{\rho_2}{\rho_1}}$$

When the initial momenta and outlet aperture areas are identical we have $dG_1^\circ = \sqrt{\rho^\circ} dG_2^\circ$, which is analogous to relationship (0.4) obtained above.

Finally, it is possible to show that the proposed hypothesis for the case of a submerged gas jet conforms in the first approximation to the assumption regarding the universality of the profile relative to the transverse component of velocity: v/u_m = idem.

This assumption together with the fact, established by many researchers (cf.[3]), of the similarity of the profiles of the longitudinal velocity component (u/u_m = idem) is equivalent to a hypothesis regarding the universality of the kinematic flow pattern in gas jets.

Let us illustrate this by the example of an axially symmetrical jet for which the flow increment in an arbitrary cross section of the submerged jet $dG = 2\pi g \rho_\infty v_\infty r dx$ is represented in dimensionless form:

$$\frac{dG^\circ}{dx^\circ} = 2\frac{r}{r_0}\frac{u_m}{u_0}\rho^\circ\frac{v_\infty}{u_m} \qquad \left(\rho^\circ = \frac{\rho_\infty}{\rho_0}\right)$$

As will be shown below [cf. formula (2.4)], the relationship

$$\frac{r}{r_0}\frac{u_m}{u_0} = \frac{1}{\sqrt{\rho^\circ}\Phi(u_m/u_0, \rho^\circ)}$$

is valid for the case under consideration, where the function Φ may be replaced by the constant $\Phi(u_m/u_0, \rho^\circ) \approx \text{const.}$ for not too high values of ρ°. Then

$$\frac{dG^\circ}{dx^\circ}\frac{1}{\sqrt{\rho^\circ}} \approx \text{const}\frac{v_\infty}{u}$$

The approximate conformity between the stated hypothesis [cf. formula (0.4)] and the assumption regarding the kinematic universality of the jets v/u_m = idem and u/u_m = idem also follow from this.

Analysis of known experimental and theoretical data of various

authors shows that the accumulation of mass along the submerged jet of incompressible fluid is satisfactorily described by the following generalized relationships:

for the main segment of an axially symmetrical jet

$$G^\circ(x^\circ) = \alpha_1(x^\circ + \beta_1) \qquad (0.5)$$

for the main segment of a plane-parallel jet

$$G^\circ(x^\circ) = \alpha_2\sqrt{x^\circ + \beta_2} \qquad (0.6)$$

for the initial segment of a planar jet

$$G^\circ(x^\circ) = \gamma x^\circ + 1 \qquad (0.7)$$

This formula may also be used for calculating the initial portion of an axially symmetrical jet in the first approximation.

In talking about the formulas describing the ejection of substance from the ambient medium by a turbulent jet, it should be noted that they are noticeably dissimilar for different authors although they are similar in structure and in their general shape satisfy relationships (0.5)-(0.7). For example, for the main segment of an axially symmetrical jet of incompressible fluid the coefficient α_1, according to the data of several authors, varies from 0.11 to 0.20, although the most frequently encountered values of α_1, according to the data in some papers [3-9], lie in the interval 0.13-0.16. The main reason for such discrepancies as well as for the lack of conformity among the formulas for determining the attenuation of the axial velocity of the jet lies, in our opinion, in the carelessness of the analysis of the experimental data. Thus authors rarely present data on the distribution of velocity and other flow parameters in the initial cross section of the jet, i.e., initial turbulence, but relate the axial velocity usually either to a maximum, or to an average (with respect to rate of flow or momentum) value of the velocity in the

initial cross section besides not stating which of these methods is being used. As will be clear from what follows, taking into account the nonuniformity of the flow at the outlet from the nozzle (characterized by the coefficients n_{1u}, n_{2u}, and n_1) introduces important corrections into the calculation, and it should necessarily be carried out when analyzing experimental data or when comparing experimental and theoretical results.

Let us present values of the coefficients in formulas (0.5)-(0.7) according to available experimental data

$$\alpha_1 = 0.155, \quad \beta_1 = -4.3, \quad \alpha_2 = 0.376, \quad \beta_2 = 0, \quad \gamma = 0.0362$$

We note that with the accumulation of experimental material and after careful analysis of already known experimental data pertaining to isothermic submerged jets under fixed conditions in the initial cross section, it may become necessary to alter these values somewhat (in particular, the coefficients in formulas (0.5)-(0.7) may prove to be dependent upon the initial degree of turbulence in the jet). Therefore to preserve the generality of future calculations we shall for the time being use the ejection laws in the form of generalized relationships (0.5)-(0.7).

It is not difficult to obtain the ejection law for an arbitrary jet from formulas (0.4)-(0.7). Thus we have for the initial segment of the jet

$$G^\circ = \sqrt{\rho^\circ}\gamma x^\circ + 1 \qquad (0.8)$$

while for the main segment of an axially symmetrical jet we obtain

$$G^\circ = \sqrt{\rho^\circ}\alpha_1 (x^\circ + \beta_1) \qquad (0.9)$$

In the derivation of these relationships the constants of integration were determined from the condition that the obtained

relationships coincide with the ejection law for an isothermic submerged jet when $\rho^{\circ} = 1$. Generally speaking, these integration constants may be functions of the initial parameters of the jet (e.g., ρ°), but here we shall consider them to be constants.

As may be noted, no quantities characterizing the kinematics of the ambient medium enter into the right sides of Eq. (0.8) and (0.9). This means that, in accordance with the adopted hypothesis, the relative velocity of the concomitant flow has no effect on the value of the acquired mass of the jet. The fact just noted is rather curious and may be of interest in problems having to do with mixture and combustion in a stream. In particular, it follows from this that mixing of the jet with the ambient medium, evidenced by the increase in the mass of gas drawn into the jet, will also occur when the velocities of the jet and concomitant flow are equal. However, if the momentum of the concomitant flow is greater than that of the jet, the flow will be determined by the characteristics of the concomitant flow, since in this case the dominant role is played by the ejection capacity (proportional, according to the adopted hypothesis, to the initial momentum) of the concomitant flow, rather than of the jet.

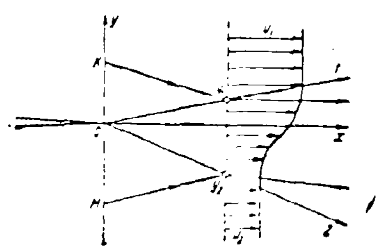


Fig. 1. Flow pattern in mixing zone of initial section of jet.

The hypothesis put forth above, expressed by Eq. (0.4), is an alternative to the equation of turbulent gas jet propagation suggested by the author together with G. N. Abramovich in an earlier monograph [3], and together with the equations of conservation of momentum and excess heat content forms a closed system for calculating a gas jet in a moving medium.

In the individual examples below we shall show what results are obtained in the practical use of the proposed hypothesis, and also a comparison will be made between these results and corresponding experimental data.

1. Initial segment of a Fluid Jet in a Concomitant Flow. Let us consider the initial segment of an isothermic turbulent jet being propagated in a moving medium.

As shown in the literature [1], on the basis of an assumption of the universality of dimensionless excess velocity profiles

$$\Delta u^\circ = \frac{u_1 - u}{u_1 - u_2} = (1 - \eta^{\frac{3}{2}})^2 \qquad \left(\eta = \frac{y - y_2}{y_1 - y_2}, \quad y_1 - y_2 = b\right)$$

it is possible to obtain from the equations of conservation of momentum, flow rate and transverse fluid equilibrium for the circuit Ky_1y_2OM (Fig. 1) the following dependence of the relative ordinate of the jet boundary layer on the parameter $m = u_2/u_1$:

$$\frac{y_1}{b} = 0.416 + 0.134m \qquad (1.1)$$

In order to solve the problem it now remains to find the variation in the thickness of the free boundary layer b along the length of the jet. For this purpose G. M. Abramovich has suggested [1] a relationship derived from L. Prandtl's theory on the mechanism of mixing in the jet. But, as noted above, this relationship gives incorrect results when $m > 0.35$. Instead of this relationship we

shall try to make use of our hypothesis on the universality of the ejection properties of a turbulent jet. For this we shall compute the amount of fluid in an arbitrary transverse cross section of the initial section of a plane-parallel jet

$$G = 2g\rho_1 u_1 (b_0 - y_1) + 2g \int_{y_1}^{y_2} \rho u dy$$

For this case $\rho^\circ = 1$ we obtain from this

$$G^\circ = \frac{G}{G_0} = 1 + \frac{b}{b_0}\left(0.55 + 0.45\, m - \frac{y_1}{b}\right) \qquad (1.2)$$

Substituting into here the function $G^\circ(x^\circ)$ according to formulas (0.8) when $\rho^\circ = 1$ and replacing the quantity y_1/b in accordance with Eq. (1.1), we obtain

$$b^\circ = \frac{\gamma}{0.134 + 0.316\, m}\, x^\circ$$

It follows from this when $\gamma = 0.0362$

$$b^\circ = \frac{0.27}{1 + 2.36\, m}\, x^\circ \qquad (1.3)$$

or

$$\bar{b} = \frac{b^\circ}{(b^\circ)_{m=0}} = \frac{1}{1 + 2.36\, m} \qquad (1.4)$$

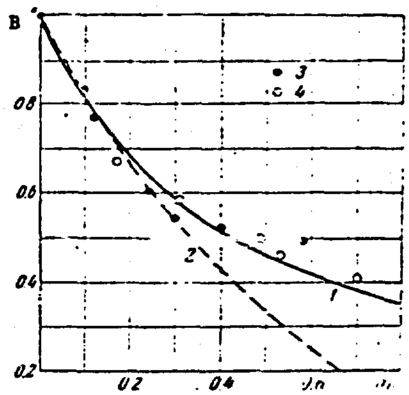


Fig. 2. Dependence of reduced thickening coefficient for the mixing zone of an isothermic jet ($\rho^\circ = 1$) on the parameter m: curve 1 is drawn according to Eq. (1.4); curve 2, according to Eq. (1.5); points 3) pertain to O. V. Yakovlevskiy's experiments [2]; points 4) to B. A. Zhestkov's experiments [4].

In Fig. 2 function (1.4) is compared with the corresponding experimental data obtained by the author [2] and by B. A. Zhestkov [4], as well as with a similar functional relationship obtained by G. N. Abramovich [1]:

$$b^* = \frac{1-m}{1+m} \qquad (1.5)$$

As follows from the comparison of the theoretical and experimental results, Eq. (1.4), which has been constructed on the basis of the adopted hypothesis, corresponds to the experimental points better than Eq. (1.5) in the range $0.35 < m < 1$, while when $m < 0.35$ both theoretical curves agree satisfactorily with the experimental values.

2. A Turbulent Jet of Heated Gas (with Variable Thermodynamic Properties) in a Concomitant Flow. Consider the main segment of a turbulent jet of heated or cooled gas issuing into a quiescent or moving medium of the same composition as the substance of the jet (Fig. 3). We note that recently in connection with the development of a different kind of system the interest in jets of intensely heated (10,000-20,000°K) gas has increased considerably.

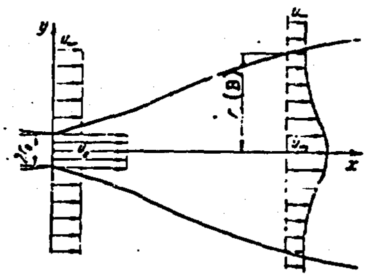


Fig. 3. Diagram of flow over the main segment of a jet being propagated in a concomitant flow.

At these temperatures physicochemical transformations

(dissociation, ionization) are observed in the gas, which lead to a significant change in its thermodynamic properties, particularly in the heat capacity, which must not be considered constant under these conditions anyway, as it usually is in the calculation of non-isothermic jets [3]. Analysis of prepared data on the thermodynamic properties of various gases, such as air [10], shows that from 0.001 to 1000 atm.(abs.) and from 1000 to 12,000°K (this limit may apparently be raised to 20,000°K) the equation of state may be written in the form

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}\left(\frac{i_0}{i}\right)^{\alpha} \qquad (2.1)$$

where i is the enthalpy, and the exponent α has the value 0.83 for air. According to preliminary calculations this exponent is of the same order of magnitude for other gases: thus for hydrogen $\alpha = 0.92$. In the future we will use the equation of state written in the form (2.1) when $P = P_0$, assuming that free turbulent jets are isobaric.

Experimental data on heated air jets [3] allows us to assume that the relative excess velocity profiles in them, just as in isothermic jets, are universal and satisfactorily described by the following equation:

$$\Delta u^{*} = \frac{\Delta u}{\Delta u_m} = \frac{u - u_\infty}{u_m - u_\infty} = (1 - \xi^{\frac{3}{2}})^2 \quad \left(\xi = \frac{y}{r}\right) \qquad (2.2)$$

In addition, we assume that the relative excess enthalpy profiles are universal, since at gas temperatures no higher than 1000°K they conform, on account of the constancy of the specific heat, to the relative excess temperature profiles of the jet, which, as we know [3] are universal. We will consider the enthalpy profiles to be described by the equation

$$\Delta i^{c} = \frac{\Delta i}{\Delta i_m} = \frac{i - i_\infty}{i_m - i_\infty} = 1 - \xi^{\frac{3}{2}} \qquad (2.3)$$

In order to determine the kinematic and thermodynamic parameters at an arbitrary point in the jet of heated gas taking into account relationships (2.2) and (2.3) it remains to find the variation of the axial velocity and enthalpy, as well as of the thickness of the jet as a function of the distance to its initial cross section. For this purpose we have available a system of three equations.

The first two equations express the laws of conservation of momentum and excess heat content in the jet and after simple transformations (cf., for example, Abramovich [3]) may be reduced to the following form:

$$\Delta u_m^\circ [m_0 A_1 + (1 - m_0) A_2 \Delta u_m^\circ] = \frac{F_0}{F} \frac{n_{2u} - m_0 n_{1u}}{1 - m_0} \tag{2.4}$$

$$\Delta i_m^\circ [m_0 B_1 + (1 - m_0) B_2 \Delta u_m^\circ] = \frac{F_0}{F} \frac{p_0 n_i - n_{1u}}{p_0 - 1} \tag{2.5}$$

Here

$$\begin{aligned}
A_1 &= \int_0^1 \frac{\rho}{\rho_{0m}} \Delta u^\circ \frac{dF}{F}, & A_2 &= \int_0^1 \frac{\rho}{\rho_{0m}} (\Delta u^\circ)^2 \frac{dF}{F} \\
B_1 &= \int_0^1 \frac{\rho}{\rho_{0m}} \Delta i^\circ \frac{dF}{F}, & B_2 &= \int_0^1 \frac{\rho}{\rho_{0m}} \Delta i^\circ \Delta u^\circ \frac{dF}{F} \\
n_{1u} &= \int_0^1 \frac{\rho_0}{\rho_{0m}} \frac{u_0}{u_{0m}} \frac{dF_0}{F_0}, & n_{2u} &= \int_0^1 \frac{\rho}{\rho_{0m}} \left(\frac{u_0}{u_{0m}}\right)^2 \frac{dF_0}{F_0} \\
n_i &= \int_0^1 \frac{\rho_0}{\rho_{0m}} \frac{u_0}{u_{0m}} \frac{i_0}{i_{0m}} \frac{dF_0}{F_0}, & m_0 &= \frac{u_\infty}{u_{0m}}, \quad p_0 = \frac{i_{0m}}{i_\infty}
\end{aligned} \tag{2.6}$$

the subscript 0 pertains to the initial cross section of the jet; ∞, to the ambient medium; and m, to the jet axis.

Let us point out that the equations written above are of a general nature and only the assumption of the universality of the profiles of Δu° and Δi° was used in their derivation. In addition these relationships are suitable for both plane-parallel and axially symmetrical jets; for these cases

$$\frac{dF}{F} = \frac{dy}{b} = d\xi, \qquad \frac{dF}{F} = 2\frac{y}{r} d\left(\frac{y}{r}\right) = 2\xi d\xi$$

respectively.

The parameters n_{1u}, n_{2u}, and n_1 introduced above characterize the nonuniformity of the velocity and enthalpy (or temperature) profile in the initial cross section of the jet; in a jet with absolutely uniform velocity and enthalpy they have the same value, equal to unity.

It should be emphasized that the difference in nonuniformity parameters from unity is often neglected in calculations of turbulent jets; as this gives results which differ significantly from actuality. This is especially true for the calculation of a jet in a concomitant flow, since, as is apparent from an analysis of Eq. (2.4), the effect of the coefficients n_{1u} and n_{2u} (when they are different from unity) increases rapidly as the parameter m_o increases from its zero value.

As an illustration we shall point out that for a steady-state turbulent velocity profile in the initial cross section of the jet (i.e., when the thickness of the boundary layer is equal to the radius of the outlet aperture) satisfying a power function with an exponent of 1/7, the parameters $n_{1u} = 0.815$, $n_{2u} = 0.680$.

We shall use Eq. (0.9) as the third relationship to close the system of equations, i.e., from now on we shall consider only an axially symmetrical jet as it is the most frequently encountered in practice. The method of calculation for a plane parallel jet is analogous and based on relationships (2.4)-(2.5) taking into account the ejection law for a plane-parallel jet described by formulas (0.4) and (0.6).

It is not difficult to show that for the assumptions regarding the universality of the profiles of Δu° and Δi° introduced above the

relative rate of gas flow in an arbitrary cross section of the jet is determined by the equation

$$G^\circ = \frac{G}{G_0} = \frac{1}{n_{1u}} \frac{F}{F_0} [m_0 A_0 + (1 - m_0) A_1 \Delta u_m^\circ] \tag{2.7}$$

Here

$$G = \int_0^F \rho u dF, \quad G_0 = \int_0^{F_0} \rho_0 u_0 dF_0, \quad A_0 = \int_0^1 \frac{\rho}{\rho_{0m}} \frac{dF}{F} \tag{2.8}$$

By solving Eqs. (2.4), (2.5), and (2.7) together we easily obtain the dependent variables in which we are interested Δu_m°, Δi_m° and F/F_0 as functions of the relative rate of flow G°, the variation of which as a function of the dimensionless distance x° is described by Eq. (0.9). We have

$$\Delta u_m^\circ \frac{m_0 A_1 + (1 - m_0) A_2 \Delta u_m^\circ}{m_0 A_0 + (1 - m_0) A_1 \Delta u_m^\circ} = \frac{1}{G^\circ} \frac{n_{2u} - m_0 n_{1u}}{n_{1u}(1 - m_0)} \tag{2.9}$$

$$\Delta i_m^\circ = k_i \Delta u_m^\circ \tag{2.10}$$

$$k_i = \frac{p_0 n_i - n_{1u}}{p_0 - 1} \frac{1 - m_0}{n_{2u} - m_0 n_{1u}} \frac{m_0 A_1 + (1 - m_0) A_2 \Delta u_m^\circ}{m_0 B_1 + (1 - m_0) B_2 \Delta u_m^\circ} \tag{2.11}$$

As will be clear from what follows, the quantity k_i varies slightly along the length of the jet, and when the velocity and enthalpy profiles are uniform in the initial cross section ($n_{1u} = n_{2u} = n_i = 1$) this quantity has a value close to that for an isothermic submerged jet $k_i = 0.753$ (in the case of a plane-parallel isothermic jet $k_i = 0.860$). Thus Eqs. (2.10) and (2.11) correspond to an almost linear relationship between the excess axial enthalpy and excess axial velocity of the jet.

To find the functions $\Delta u_m^\circ(x^\circ)$ and $\Delta i_m^\circ(x^\circ)$ it is required to know the values of A_i and B_i. Since the profiles of Δu° and Δi° in transverse cross sections are known as functions of ξ, while the quantity ρ/ρ_{0m} when (2.1) is used as the equation of state may be represented in the form

$$\frac{\rho}{\rho_{0m}} = \frac{p_0^{\alpha}}{[1+(p_0-1)\,\Delta i^{\circ}\Delta i_m^{\circ}]^{\alpha}} \tag{2.12}$$

after integration it is possible to find the coefficients A_i and B_i as functions of the quantities p_0 and Δi_m°. But, for the fractional values of α given above, the integrals in Eqs. (2.6) do not yield elementary functions when relationship (2.12) is substituted into them, and therefore numerical calculations were made for their determination in the range of variation of the quantity $(p_0 - 1)\Delta i_m^{\circ}$ from 0.75 to 20.0 when $0.5 \leq \alpha < 1$. An empirical analysis of the results of these calculations has shown that in said range the values of A_i and B_i can with an accuracy sufficient for practice be computed using the following formulas:

$$A_0 = \frac{p_0^{\alpha}}{[1+0.280\,(p_0-1)\,\Delta i_m^{\circ}]^{\alpha}}, \qquad A_1 = \frac{0.257\,p_0^{\alpha}}{[1+0.650\,(p_0-1)\,\Delta i_m^{\circ}]^{\alpha}}$$

$$A_2 = \frac{0.134\,p_0^{\alpha}}{[1+0.760\,(p_0-1)\,\Delta i_m^{\circ}]^{\alpha}} \tag{2.13}$$

$$B_1 = \frac{0.428\,p_0^{\alpha}}{[1+0.500\,(p_0-1)\,\Delta i_m^{\circ}]^{\alpha}}, \qquad B_2 = \frac{0.178\,p_0^{\alpha}}{[1+0.730\,(p_0-1)\,\Delta i_m^{\circ}]^{\alpha}}$$

Substitution of relationships (2.13) into Eq. (2.9) by taking formula (2.10) into account and replacement of the quantity G° according to formula (0.9) using the obvious equality

$$\rho^{\circ} = p_0^{\alpha} \tag{2.14}$$

allows us to obtain the relationship between the excess axial velocity and the distance from the nozzle in final form

$$\frac{\Delta u_m^{\circ}}{\psi_{12}}\;\frac{1.92\,\psi_{12}\mu_0 + \Delta u_m^{\circ}}{3.895\,\psi_{01}\mu_0 + \Delta u_m^{\circ}} = \frac{1.92\,\nu_u}{p_0^{\frac{\alpha}{2}}\,a_1\,(x^{\circ}+\beta_1)} \tag{2.15}$$

Here

$$\psi_{01} = \left[\frac{1+0.650\,(p_0-1)\,k_i\Delta u_m^{\circ}}{1+0.280\,(p_0-1)\,k_i\Delta u_m^{\circ}}\right]^{\alpha}, \qquad \psi_{12} = \left[\frac{1+0.760\,(p_0-1)\,k_i\Delta u_m^{\circ}}{1+0.650\,(p_0-1)\,k_i\Delta u_m^{\circ}}\right]^{\alpha}$$

$$\mu_0 = \frac{m_0}{1-m_0}, \qquad \nu_u = \frac{n_{2u} - m_0 n_{1u}}{n_{1u}\,(1-m_0)} \tag{2.16}$$

The variation in the enthalpy Δi_m° along the axis of the jet

is determined with the aid of Eq. (2.10). The coefficient k_1 contained in the theoretical equations may be determined in the first approximation using formula (2.11), into which are substituted the quantities A_1, A_2, B_1, B_2 computed from relationships (2.13) using the approximation

$$\Delta i_m^\circ = 0.753 \frac{p_0 n_1 - n_{1w}}{p_0 - 1} \frac{1 - m_0}{n_{2w} - m_0 n_{1w}} \Delta u_m^\circ \tag{2.17}$$

As calculations show, the values of k_1 determined in this way are extremely close to the values of k_1 found in the second approximation.

The axial temperature of the jet T_m for a particular value of Δi_m° is determined from tables of thermodynamic functions for the given gas (in the case of air - the tables cited at the end of the article [10]) as a function $T_m = T_m(i_m)$, where

$$i_m = (i_0 - i_\infty)\,\Delta i_m^\circ + i_\infty \tag{2.18}$$

The thickness of the jet ($r^\circ = r/r_0 = \sqrt{F/F_0}$) in an arbitrary cross section is found from Eq. (2.4) using the function $\Delta u_m^\circ(x^\circ)$, determined by Eq. (2.15).

Thus the equations obtained above permit the variation in axial velocity and temperature and also the thickness of the jet to be determined as functions of the dimensionless coordinate x° in terms of the parameters of the jet and concomitant flow in the initial cross section. The value of the velocity and enthalpy at an arbitrary point in the jet is determined from their known values at the axis with the aid of Eqs. (2.2) and (2.3).

In order to appraise the effect of the parameter p_0, which characterizes the difference in the thermodynamic properties of the jet and the ambient medium, on the propagation of the jet, let us

consider the simplest case of the escape of a heated gas into a quiescent medium ($m_0 = 0$). In this case from Eq. (2.15) when $\mu_0 = 0$ we obtain the following formula for the axial velocity of the jet after replacing the coefficients α_1 and β_1 by their respective numerical values:

$$u_m^\circ \left[\frac{1 + 0.650\,(p_0 - 1)\,k_1 u_m^\circ}{1 + 0.700\,(p_0 - 1)\,k_1 u_m^\circ} \right]^2 = \frac{12.4\,\nu_u}{(x^\circ - 4.3)\sqrt{\rho_0^\circ}} \tag{2.19}$$

where

$$k_1 = 0.753\,\frac{p_0 n_1 - n_{1u}}{n_{2u}\,(p_0 - 1)} \left[\frac{1 + 0.730\,(p_0 - 1)\,k_1 u_m^\circ}{1 + 0.700\,(p_0 - 1)\,k_1 u_m^\circ} \right]^2 \tag{2.20}$$

or approximately

$$k_1 \approx 0.753\,\frac{p_0 n_1 - n_{1u}}{n_{2u}\,(p_0 - 1)} \tag{2.21}$$

From an analysis of the law for the damping of the axial velocity of an isothermic jet (2.19) it follows that in a jet heated relative to the ambient medium the axial velocity drops off considerably faster than in an isothermic jet, with the value of u_m° for a given value of x° being inversely proportional to the square root of the ratio of the areas of the ambient medium and jet. As an illustration the calculated functions $u_m^\circ(x^\circ)$ are presented in Fig. 4 for an isothermic ($p_0 = 1$, or $\rho^\circ = 1$) and an intensely heated ($p_0 = 26$, or $\rho^\circ = 15$) jet of air escaping into a quiescent medium.

Also presented are experimental points characterizing the axial velocity in an intensely heated air jet according to data obtained by V. Ya. Bezmenov and V. S. Borisov.* As is apparent from the results in Fig. 4, theoretical formula (2.19) obtained on the basis of the hypothesis regarding the universality of the ejection properties

*The data of this unpublished work was obligingly made available to the author for the preparation of the present article.

of turbulent jets agrees satisfactorily with the experimental data right up to very high values of the parameter p_o.

The damping curve for the excess axial temperature ΔT_m° in a non-isothermic jet is also presented in Fig. 4, and the corresponding experimental values, which, as can be seen, are close to the theoretical, are also shown there. This fact, in our opinion, attests the negligibly small effect of radiant heat transfer, which has not been taken into account in the present calculations, on the thermodynamic properties of intensely heated turbulent jets.

The theoretical and experimental values of the thickness of a jet of intensely heated air are compared in Fig. 5, and there is also presented a curve characterizing the variation in the thickness of an isothermic jet along its length. It is apparent that the theoretical thickness of a heated jet which conforms well to experimental data markedly exceeds the thickness of an isothermic jet for the same values of x°.

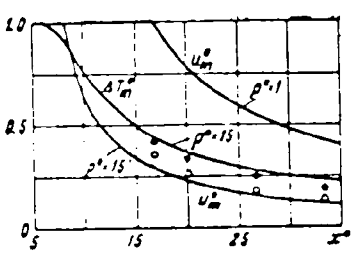


Fig. 4. Variation in the axial velocity u_m° along an isothermic ($\rho^\circ = 1$) and a heated ($\rho^\circ = 15$) air jet; curves are calculated; small circles pertain to experiments performed by V. Ya. Bezmenov and V. S. Borisov for $\rho^\circ = 15$; variation in axial temperature ΔT_m° along the axis of a heated ($\rho^\circ = 15$) air jet; curve is calculated; points pertain to experiments at $\rho^\circ = 15$.

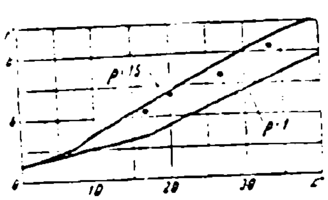


Fig. 5. Increase in the thickness of an isothermic ($\rho^\circ = 1$) and a heated ($\rho^\circ = 15$) air jet along the x-axis; curves are calculated, points pertain to experiments at $\rho^\circ = 15$.

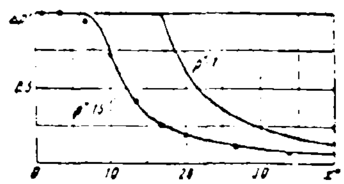


Fig. 6. Variation in the velocity heat $\Delta p^{\circ} = \rho_m u_m^2/\rho_0 u_0^2$ along the axis of heated ($\rho^{\circ} = 15$) and isothermic ($\rho^{\circ} = 1$) air jets; curves are calculated, points pertain to experiments at $\rho^{\circ} = 15$.

The theoretical and experimental data presented in Fig. 5 permits us to draw the conclusion that, in spite of the difference in thicknesses of isothermic and heated submerged jets, the angle between the boundary of the main section of the jet and the x-axis, apparently does not depend on the parameter ρ°, and its tangent has a constant value equal to 0.22.

Finally, in Fig. 6 there is shown a theoretical curve characterizing the attenuation of the velocity head along the axis of a heated ($\rho^{\circ} = 15$) air jet, and experimental points obtained by V. Ya. Bezmenov and V. S. Borisov are plotted there also. A theoretical curve for the axial velocity head in an isothermic air jet is shown for comparison in the same figure. As we see, the coincidence between the theoretical and experimental results is entirely satisfactory for the case under consideration ($\rho^{\circ} = 15$).

A comparison of data concerning the velocity head at the jet axis for $\rho^{\circ} = 1$ and $\rho^{\circ} = 15$ does not speak well for L. A. Vulis' hypothesis [11] regarding the universality of velocity head distribution in gas jets.

Theoretical relationship (2.19) satisfies the experimental data

well also for small values of ρ°. It must be noted however than when $\rho^\circ \leq 3$ (i.e., when the temperature of the jet does not exceed 1000°K) the equation of state must be used in its usual form

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}\frac{T_0}{T}$$

and the specific heat c_p must be considered constant. Then in all relationships obtained above it is necessary to set $\alpha = 1$ and, consequently, $p_0 = \rho^\circ$. Calculations performed by this method for a warmed ($\rho^\circ = 2$), submerged ($m_0 = 0$) jet are compared with the corresponding experimental data [7] in Fig. 7. As we see, a quite distinct coincidence between the experimental and theoretical results is observed in this case.

A comparison of curves for the attenuation of the excess axial velocity of a non-isothermic jet ($\rho^\circ = 2.04$) propagating in a concomitant flow ($0 < m_0 < 0.5$) with experimental data [12] obtained for analogous conditions is presented in Fig. 8, where n_{2u}/n_{1u} is taken equal to 0.95 for all values of m_0. It follows from an analysis of this figure that the hypothesis regarding the universality of the ejection properties of turbulent jets is justified in the general case of the propagation of a non-isothermic jet in a moving medium.

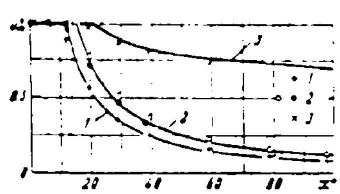


Fig. 7. A comparison of theoretical and experimental data on the axial velocity of a warmed up ($\rho^\circ = 2$) and an isothermic ($\rho^\circ = 1$) jet in a concomitant flow ($n_{1u} = 0.888$ and $n_{2n} = 0.806$); curves are calculated; points pertain to experiments conducted by Yu. V. Ivanov and Kh. N. Suy [7] for the values: 1) $\rho^\circ = 2$, $m_0 = 0$; 2) $\rho^\circ = 1$, $m_0 = 0$; 3) $\rho^\circ = 1$, $m_0 = 0.65$.

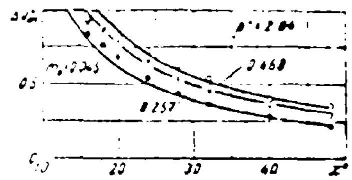


Fig. 8. Variation of the excess axial velocity Δu°_m in a non-isothermic ($\rho^\circ = 2.04$) jet propagating in a concomitant flow ($n_{2u}/n_{1u} = 0.95$); curves are calculated, points pertain to experiments conducted by O. Pabst [12].

3. A Jet of Incompressible Fluid in a Concomitant Flow. In order to more clearly isolate the role of the parameter m_0 in the propagation of turbulent jets, let us consider the special case of an isothermic ($\rho^\circ = 1$) jet issuing into a fluid flow parallel to it. Under these conditions ($\rho^\circ = 1$) using formulas (2.2) and (2.6) we obtain

$$A_0 = 1, \ A_1 = 0.257, \ A_2 = 0.134 \qquad (3.1)$$

and the final equation determining the law for the attenuation of the axial velocity assumes the form

$$\Delta u^\circ_m \frac{1.92\,\mu_0 + \Delta u^\circ_m}{3.86\,\mu_0 - \Delta u^\circ_m} = \frac{12.4\,v_0}{x^\circ - 4.3} \qquad (3.2)$$

The thickness of the jet ($r^\circ = r/r_0 = \sqrt{F/F_0}$) for a given value of x° is found from equation (2.4), the values of the coefficients entering into it are determined by equalities (3.1).

In order to illustrate the characteristics of the spreading of an isothermic fluid jet in a concomitant flow (when $n_{1u} = n_{2u} = 1$) curves of the attenuation of the excess axial velocity Δu°_m and of the increase in the thickness of the jet r° when $0 \leq m_0 \leq 1$. As also

follows from G. N. Abramovich's theory [3], the presence of concomitant flow leads to an increase in the long range nature of the jet, but as the parameter m_0 approaches unity the displacement of the axial velocity attenuation curve becomes weaker and weaker until the curve finally reaches its limiting position when $m_0 = 1$. Moreover, the thickness of the jet increases along its length in this case, just as when $m_0 \neq 1$, whereas according to a theory put forth in the literature [3] the jet does not mix with the ambient medium when $m_0 = 1$.

A comparison of the theoretical values of the axial velocity of an isothermic jet in a concomitant flow ($0 < m_0 < 1$) with the corresponding experimental data taken from the literature [7], is given in Fig. 7. Moreover, we have taken into account the initial non-uniformity of the jet, which according to Yu. V. Ivanov's supplementary information, is in keeping with $n_{1u} = 0.888$ and $n_{2u} = 0.806$. As we see, the hypothesis regarding the universality of the ejection properties of turbulent jets is also correct for the spreading of an isothermic jet in a concomitant flow the velocity of which does not exceed the initial velocity of the jet.

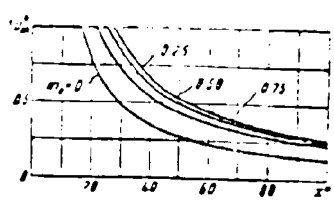


Fig. 9. The effect of the parameter m_0 on the attenuation of the axial velocity of a jet of incompressible fluid in a concomitant flow ($n_{1u} = n_{2u} = 1$).

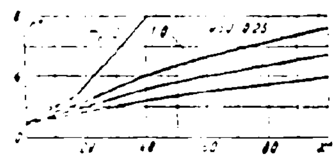


Fig. 10. The effect of the parameter m_0 on the thickness of a jet of incompressible fluid in a concomitant flow ($n_{1u} = n_{2u} = 1$).

4. A Supersonic Jet in the Design and Off-Design Efflux Regimes. Let us stop to consider the case of a submerged supersonic jet ($m_0 = 0$). The propagation of a supersonic jet in a concomitant flow can be analyzed in a similar way. The momentum equation may be presented in the form [3]

$$u_m^\circ A_2 = \frac{F_0}{F}\left[n_{2u}n + \frac{1 - a\lambda_0^2}{a\lambda_0^2}(n - 1)\frac{a}{1 + a}\right]$$

$$\lambda_0 = \frac{u_0}{a_*}, \quad a = \frac{k - 1}{k + 1}, \quad n = \frac{P_0^*}{P^*} \approx \frac{P_0}{P_\infty}, \quad k = \frac{c_p}{c_v} \qquad (4.1)$$

Here P_0^* and P^* are the actual and design pressures in the receiver; λ_0 is the velocity ratio; a_* is the critical sound velocity; and c_p and c_v are the specific heats. The quantity A_2 depends upon the value of $a\lambda_0^2$ and the axial velocity u_m°; this functional relationship was presented in the literature [3].

The relative flow of the gas in an arbitrary cross section of the jet is expressed by the formula

$$G^\circ = \frac{F}{F_0}\frac{A_1}{n_{1u}n}u_m^\circ \qquad (4.2)$$

Here the coefficient A_1 is a known [3] function of $a\lambda_0^2$ and u_m°. By combining relationships (4.1) and (4.2) we obtain:

$$u_m^\circ = \frac{A_2}{A_1}\frac{1}{n_{1u}G^\circ}\left[n_{2u} + \frac{1 - a\lambda_0^2}{a\lambda_0^2}\left(1 - \frac{1}{n}\right)\frac{a}{1 + a}\right] \qquad (4.3)$$

If we substitute into here the function $G^\circ = G^\circ(x^\circ, o^\circ)$ according to formula (0.9), where

$$\rho^\circ = \frac{T_0^\circ}{T_\infty^\circ}\,\frac{1 - a\lambda_0^2}{n} \tag{4.4}$$

and introduce the notation

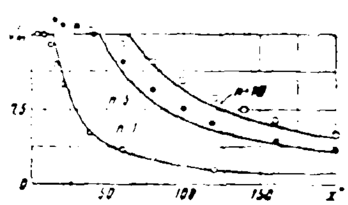
$$\frac{A_1}{A_2} = 1.92\,S\,(u_m^\circ, a\lambda_0^2) \tag{4.5}$$

formula (4.3) assumes the form

$$u_m^\circ = \frac{1}{n_{1u}}\left[n_{2u} + \frac{1 - a\lambda_0^2}{a\lambda_0^2}\left(1 - \frac{1}{n}\right)\frac{a}{1+a}\right]\frac{12.4\,S\,(u_m^\circ, a\lambda_0^2)}{\sqrt{\rho^\circ}\,(x^\circ - 4.3)} \tag{4.6}$$

In the first approximation the function $S(u_m^\circ, a\lambda_0^2)$ when $0 \le a\lambda_0^2 \le 0.7$ and $u_m^\circ \le 1$ may with an accuracy acceptable for practice be considered as a constant $S(u_m^\circ, a\lambda_0^2) \approx 1$.

The theoretical curves for the attenuation of the axial velocity i supersonic air jets when $a\lambda_0^2 = 0.311 (M_0 = 1.5)$ is presented in Fig. 11 for several values of the off-design parameter <u>n</u>, while in Fig. 12 theoretical curves are given for design efflux with supersonic velocity when $a\lambda_0^2 = 0.644$ $(M_0 = 3.0)$



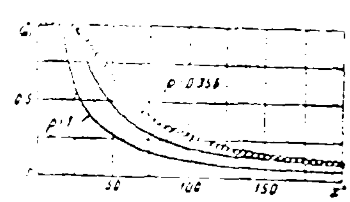


Fig. 11. Theoretical and experimental data on the velocity variation at the axis of supersonic air jets ($M_0 = 1.5$) for design and off-design efflux regimes; curves are theoretical, points pertain to the experiments conducted by B. A. Zhestkov et al. [3].

Fig. 12. Theoretical and experimental data on the velocity variation at the axis of supersonic ($M_0 = 3.0$, $\rho^\circ = 0.356$) air jets: curves are theoretical; points pertain to experiments conducted by B. A Zhestkov et al. [3].

Also presented here are the corresponding experimental points borrowed

from the papers published by B. A. Zhestkov et. al. (cf.[3]). A curve of the attenuation of axial velocity for an isothermic air jet with small initial velocity ($M_0 \approx 0$, $\rho^\circ = 1$) is presented in Fig. 12 for the sake of comparison. As we see the supersonic jet is of considerably longer range, i.e., for a given value of x° its axial velocity u_m° is higher than that of a jet with small initial velocity. Analysis of this material indicates that the calculations made on the basis of the hypothesis about the universality of the ejection properties of turbulent jets also agree satisfactorily with known experimental data for supersonic (including off-design) air jets right up to a gas velocity corresponding to a Mach number $M_0 = 3$.

5. Gas Liquid Jet. In conclusion we shall consider the efflux of a gas jet into a medium filled with atomized liquid. The theory of such a jet, developed on the basis of the author's formula

$$\left(\frac{db}{dx}\right)_{x=0} \sim u_0 \int_0^b \rho dy \Big/ \int_0^b \rho u dy \qquad (5.1)$$

was first published in Abramovich [3].

We shall try to apply our hypothesis in this case as well. As has been shown in the aforementioned article [3], the momentum equation leads to the following relationship between the axial velocity and the area of a lateral cross section of the gas-liquid jet:

$$u_m^\circ = N \frac{F_0}{F} \qquad (5.2)$$

where N = 3.85 for an axially symmetrical jet. From the equation for the conservation of the initial amount of gas we obtain the following relationship between the gas concentration at the axis of the jet and the axial velocity:

$$u_m^\circ = K \varkappa_{gm} \qquad (5.3)$$

In addition, it is not difficult to show that the flow rate of the gas-liquid mixture in an arbitrary cross section may be expressed by the formula

$$G^\circ = 0.428 \frac{F}{F_0} \frac{u^\circ_m}{\varkappa_{gm}} \tag{5.4}$$

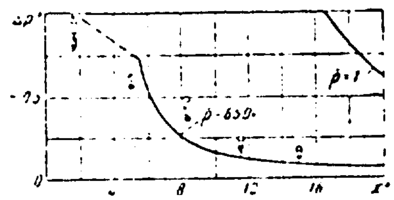


Fig. 13. A comparison of the theoretical curves for the attenuation of the axial velocity head $\Delta p^\circ = \rho_m u_m^2 / \rho_s u_s^2$ in an air jet issuing into water ($\rho^\circ \approx 650$), and in a jet with constant density ($\rho^\circ = 1$) with experimental data for a gas-liquid jet [13].

If we substitute into here G° as a function of x° according to formula (0.9), relationships (5.2) and (5.4) form a closed system of equations, which permits us to determine the velocity and concentration of the mixture at the axis of the jet, and also the jet's thickness.

In some cases the dynamic properties of a gas-liquid jet, determined by its velocity head, are of interest. As has been shown in the literature [3], the velocity head of the mixture at the axis of the main segment of the jet, relative to the initial velocity head of an air jet may be expressed by the following approximate relationship:

$$\frac{\rho_m u_m^2}{\rho_s u_s^2} \approx K u_m^\circ \tag{5.5}$$

Substituting here u_m° as a function of x°, determined by relationships (5.2) and (5.4), we finally obtain for an efflux of air into water with a high subsonic velocity ($\rho^\circ \approx 650$)

$$\frac{\rho_m u_m^2}{\rho_0 u_0^2} = \frac{0.92}{x^\circ - 4.3} \qquad (5.6)$$

A comparison of the theoretical curve for the attenuation of the velocity head along the axis of the jet according to formula (5.6) with experimental data borrowed from the literature [13] shows (Fig. 13) acceptable agreement between the method of calculation developed here and experiment, especially if the considerable spread of the experimental points and the approximate nature of relationship (5.5) are taken into account. The dotted line in Fig. 13 joins the curves for the attenuation of the velocity head in the initial ($x^\circ \approx 2$) and main ($x^\circ > 5.5$) segments of the jet and is drawn ideally.

A comparison of data on the variation in the velocity head along the axis of a gas-liquid jet with the corresponding curve for a submerged turbulent jet with constant density ($\rho^\circ = 1$), just as in the case of jets of intensely heated air, shows L. A. Vulis' assumption [11] on the universality of velocity head fields (flow of momentum) in turbulent jets to be invalid.

It must be emphasized that in the case of an isothermic jet, propagating in a concomitant flow, the results of a calculation based on the hypothesis regarding the universality of ejection properties when $m_0 < 0.35$ is very close to theoretical data obtained on the basis of G. N. Abramovich's theory [3]. Moreover, both the one and the other set of results agrees well with experimental material. However, at the same time that G. N. Abramovich's theory, for the reasons pointed out above, begins to differ noticeably from the experimental data as the value of m_0 approaches unity, the hypothesis regarding the universality of the jet's ejection properties entirely agrees with experiment even in this range of variation of the parameter m_0.

We add that the results of calculation according to the proposed method when $m_0 < 0.35$ agrees satisfactorily with the theory of gas jets, developed by the author and published in monograph form [3]. This is true both for jets issuing from the nozzle at high velocity and for gas-liquid jets. However, in the theory just mentioned non-isothermic gas jets are considered under the assumption that the specific heat c_p is constant, which is not true when the temperature of the gas is considerable ($c^\circ > 3$). If the dependence of c_p on temperature is taken into account in the theoretical formulas, as was done in the present work, the above-noted conformity between the calculations using the new method and using the gas jet theory [3] in which the widening of the jet is determined from the relationship

$$\frac{db}{dx} \sim \frac{u_1 - u_2}{u_{cp}} \qquad \left(u_{cp} = \int_0^b \rho u dy \Big/ \int_0^b \rho dy \right)$$

will probably also be observed in the case of strongly non-isothermic gas jets.

If we take into account the fact that the hypothesis regarding the universality of ejection properties agrees satisfactorily with the results of experimental investigations of jets with a essentially variable density (plasma, gas-liquid, and supersonic jets) then the method of calculation based on this hypothesis may be recognized as being sufficiently reliable.

Analogous problems for plane-parallel gas jets may also be solved by the method proposed here. In addition the method of calculation based on the hypothesis regarding the universality of the ejection properties of turbulent gas jets will apparently also prove applicable to the case of jets propagating in a space bounded by solid walls (e.g., in the mixing chamber of a jet pump) in analyzing

gas jets growing in a drifting side flow as well as in examining the problems of turbulent combustion in a flow.

In conclusion the author takes the opportunity to thank G. N. Abramovich for his extremely useful advice in the completion of this work, and also to express his gratitude to Ye. Ya. Firsova who participated in the calculations.

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REFERENCES

1. G. N. Abramovich. O turbulentnom smeshenii na granitse dvukh ploskoparallel'nikh potokov zhidkosti (pri sputnom i vstrechnom dvizhenii), Sb. "Teoreticheskaya gidromekhanika", No. 19, pod red. L. I. Sedova. Oborongiz, 1956.

2. O. V. Yakovlevskiy. K voprosy o tolshchine zony turbulentnogo peremeshivaniya na granitse dvukh potokov gaza raznoy skorosti i plotnosti, Izv. AN SSSR, OTN, No. 10, 1958.

3. G. N. Abramovich. Teoriya turbulentnykh struy, Fizmatgiz, 1960.

4. G. N. Abramovich. Turbulentnaya struya v dvizhushcheysya srede, Izv. AN SSSR, OTN, No. 6, 1957.

5. G. N. Abramovich. Turbulentnyye svobodnyye strui zhidkostey i gazov, Gosenergoizdat, 1948.

6. V. N. Taliyev. Aerodinamika ventilyatsii, Stroyizdat, 1954.

7. Yu. V. Ivanov i Kh. N. Suy. Issledovaniye razvitiya strui v sputnom potoke, Izv. AN EstSSR, seriya tekhn. i fiz.-mat. n., 7, No. 2, 1958.

8. M. L. Albertson, J. B. Day and R. A. Jensen. Diffusion of submerged jets. Proc. of the ASCE, 1948, v. 74, pp. 1571-1596.

9. J. P. Taylor, H. L. Grimmett and E. W. Comings. Isothermal free jets of air mixing with air. Chem. Eng. Progr., 1951, v. 74, No. 4, pp. 175-180.

10. A. S. Predvoditelev, Ye. V. Stupochenko, Ye. V. Samuylov, I. P. Stakhanov, A. S. Pleshanov and I. B. Rozhdestvenskiy. Tablitsy termodinamicheskikh funktsiy vozdukha v intervale temperatur ot 1,000 do 12,000°K i intervale davleniy ot 0.001 do 1000 atm., Izd-vo AN SSSR, 1957.

11. L. A. Vulis. O turbulentnykh gazovykh struyakh, Izv. AN KazSSR, seriya energetich., 1954, vyp.6.

12. O. Pabst. Die Ausbreitung heisser Gasstrahlen in bewegter Luft, II Teil. Deutsche Luftfahrforschung, U. u. M. 8007, 1944.

13. B. F. Glikman. O struye gaza v zhidkosti, Izv. AN SSSR, OTN, Energetika i avtomatika, No. 2, 1959.

MIXING TURBULENT JETS OF DIFFERENT DENSITY

G. N. Abramovich

A number of papers [1-4] have dealt with the problem of the turbulent mixing of jets of different density. However, the contradictory results of the theoretical papers of different authors and the significant discrepancies in a number of cases between the theory and experiments force us to return again to this problem. Until recently the following two cardinal questions in jet theory had remained vague:

1) Are the same dimensionless velocity profiles retained in a gas jet as in a jet of incompressible fluid?

2) How does one density field or another influence the shape of the jet boundary?

The material presented in the author's monograph [3] and in articles by Vulis [4], Yershin [5], and Glikman [6] show that the first question may be answered in the affirmative, i.e., it is possible to consider the velocity profile in different cross sections of a jet of essentially variable density (including a combustion torch and a two-phase gas-liquid jet) to be expressed by the same universal law as in the case of an incompressible fluid.

The same may be said concerning the universality of the profile of the stagnation temperature and additive concentration for a jet of moderate velocity (subsonic or low supersonic) and for a moderate temperature difference; the problem of the temperature profile in a jet of high supersonic velocity or when the difference in the temperatures at the boundaries of the mixing zone is great requires further experimental study.

The results pertaining to the second question posed above are less distinct.

In the author's paper [1] the thickness of the mixing zone of two jets of different density was evaluated on the assumption that the degree of turbulence in a compressible gas is characterized by the ratio of the velocity head of the pulsation velocity to the velocity head of the average velocity.

Subsequent experiments have shown that the evaluation of the influence of compressibility on the thickness of the jet obtained in this case is not even reliable with respect to the sign in a number of cases (for example, according to this theory it turns out that as the density increases in a submerged jet the jet's thickness increases, while the opposite effect is observed in experiments [3]).

In O. V. Yakovlevskiy's paper [2] another method was suggested for taking into account the influence of the compressibility of the medium on the thickness of the jet, which method agrees satisfactorily with experimental data when the degree of compressibility for submerged jets is not very great, and also in cases when the velocity heads in the mixing jets are considerably different. In the authors monograph [3] O. V. Yakovlevskiy's method was extended for large density gradients.

However this method of taking into account the compressibility also proves not accurate enough for very high degrees of compressibility (plasma jets) and altogether unsuitable for the mixing of jets in which the velocity heads are commensurate.

It follows from theoretical considerations that the instability of the tangential discontinuity surface, leading to turbulence, is due to the difference in the velocity heads on either side of this surface, i.e., when the velocity heads are the same the discontinuity surface is most stable and, consequently, the mixing zone has minimum thickness in this case.

Experiments performed by L. A. Vulis [4] and I. B. Palatnik [7] bear out these considerations by showing that regardless of the difference in the densities and velocities of the adjacent jets their least deterioration is observed when the velocity heads are equal. O. V. Yakovlevskiy's method of accounting for compressibility [2] is found to be in contradiction with these results.

Proposed below is a new method for taking into account the influence of the compressibility on the thickness of jet mixing zones, which does not contradict the experimental data for any of the possible mixing regimes.

As in all the above-mentioned methods, we will consider the rate of increase in the mixing zone along its length to be proportional to the degree of flow turbulence

$$\frac{db}{dx} \sim \varepsilon = \frac{|v'|}{|u|} \tag{1}$$

Furthermore, we may assume that the momentum acquired (or lost) by the instantaneous mass of a jet in connection with turbulent pulsations is proportional to the difference in the velocity heads at a

distance equal to the mixing length

$$(\rho u)\,|v'| \sim l\,\frac{d(\rho u^2)}{dy} \tag{2}$$

Here ρu is the local value of the flow density of the forward (along the x-axis) stream; $|v'|$, the average value of the lateral pulsation velocity (along the y-axis); l, the mean value of the mixing length; and $\underline{b}$, the thickness of the mixing zone at a distance $\underline{x}$ from the beginning of mixing.

But the lateral velocity head gradient is proportional to the difference in the velocity heads at the boundaries of the mixing zone and inversely proportional to the zone's thickness;

$$\frac{d(\rho u^2)}{dy} \sim \frac{\rho_1 u_1^2 - \rho_2 u_2^2}{b}$$

Hence

$$\frac{|v'|}{(u)} \sim \frac{l}{b}\,\frac{\rho_1 u_1^2 - \rho_2 u_2^2}{(\rho u)(u)} \tag{3}$$

Subscripts 1 and 2 pertain to the two mixing jets.

It is necessary to give a definition of the characteristic values of the flow density (ρu) and velocity (u), which must be substituted into the denominator of the right side of the last expression. We shall assume that some velocity, which is averaged for the zone (characteristic velocity) is defined in the following way:

$$\frac{(d)}{(\pi d)} = (u)$$

while the characteristic values of the density of the flow and the density of the medium are obtained by averaging these quantities with respect to the thickness of the mixing zone

$$(\rho u) = \frac{1}{b}\int_0^b \rho u\,dy, \qquad (\rho) = \frac{1}{b}\int_0^b \rho\,dy$$

Then if the relative values of the mixing length are universal in different cross sections of the mixing zone $(l/b) = \text{idem}$) we have

$$\frac{db}{dx} \sim \frac{\rho_1 u_1^2 - \rho_2 u_2^2}{(\rho u)^2}(\rho)$$

$$\frac{db}{dx} = \frac{cb}{4}(\rho_1 u_1^2 - \rho_2 u_2^2) \int_0^b \rho dy \Big/ \left(\int_0^b \rho u dy\right)^2 \tag{4}$$

The values of the integrals are calculated for given velocity and density profiles [3]. The four was inserted into the denominator of the right side so that the coefficient of proportionality (c) would have the same value as in the author's monograph [3] for the special cases of the mixing of jets of incompressible fluid and for submerged gas jets. If the density in the jets does not differ greatly (by less than a factor of two), then it is possible to assume

$$\frac{1}{b}\int_0^b \rho dy \approx \frac{\rho_1 + \rho_2}{2}, \qquad \frac{1}{b}\int_0^b \rho u dy \approx \frac{\rho_1 u_1 + \rho_2 u_2}{2}$$

Then

$$\frac{db}{dx_4} = \frac{c}{2}\,\frac{\rho_1 u_1^2 - \rho_2 u_2^2}{(\rho_1 u_1 + \rho_2 u_2)^2}(\rho_1 + \rho_2) = c\,\frac{1 - m^2\rho^\circ}{(1 + m\rho^\circ)^2}\,\frac{1 + \rho^\circ}{2} \tag{5}$$

Here

$$m = \frac{u_2}{u_1}, \qquad \rho^\circ = \frac{\rho_2}{\rho_1}$$

In a jet of incompressible fluid ($\rho^\circ = 1$) this new expression reduces to the expression previously [3] recommended by the author

$$\frac{db}{dx} = c\frac{1 - m}{1 + m}$$

In the case of concomitant jets of equal velocity ($m = 1$) we have from (5)

$$\frac{db}{dx} = \frac{c}{2}\,\frac{1 - \rho^\circ}{1 + \rho^\circ}$$

For a submerged jet ($m = 0$) when the degree of compressibility is not very great, we obtain from (5) the same expression as was obtained in the literature [2]

$$\frac{db}{dx} = c\,\frac{1 + \rho^\circ}{2}$$

However, when the difference in the density of the jet and the ambient medium is large the new expression for the thickness of the mixing zone differs significantly from the old one [3]. For a submerged jet ($m = 0$) it follows from (4)

$$\frac{db}{dx} = \frac{c}{4}\int_0^b \frac{\rho}{\rho_1}\frac{dy}{b} \Big/ \left(\int_0^b \frac{\rho}{\rho_1}\frac{u}{u_1}\frac{dy}{b}\right)^2 \tag{6}$$

whereas the previous expression had the form

$$\frac{db}{dx} = \frac{c}{2}\int_0^b \frac{\rho}{\rho_1}\frac{dy}{b} \Big/ \int_0^b \frac{\rho}{\rho_1}\frac{u}{u_1}\frac{dy}{b} \tag{7}$$

The integrals in both of these expressions have been computed for any value of ρ^0 in chapter 7 of the author's above-mentioned monograph [3]. For example, when $\rho^0 = 10$ the mixing zone according to new expression (6) turns out to be almost twice as thick as by former expression (7), which agrees with the experimental data.

For equal velocity heads ($m^2\rho^0 = 1$) there is no mixing ($db/dx = 0$) in accordance with (5). This can be true only when the initial turbulence in the jets is small; if such turbulence is sufficiently high, then for values of $m^2\rho^0$ close to unity the mixing is determined not by the difference in the velocity heads of the jets but by their initial turbulence. The proposed new method for accounting for the effect of the compressibility of the medium on the thickness of the mixing zone requires careful experimental verification and, possibly, further refinement. However the fact that this method agrees with all available experimental data in the limiting cases of a submerged gas jet, of mixing of jets of incompressible fluid, and of mixing of jets of different density but nearly equal velocity heads speaks well for it.

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REFERENCES

1. G. N. Abramovich. O tolshchine zony turbulentnogo smesheniya na granitse dvukh struy raznoy skorosti, temperatury, i molekulyarnogo vesa. AN SSSR, OTN, No. 3, 1957.

2. O. V. Yakovlevskiy. K voprosu o tolshchine zony turbulentnogo smesheniya na granitse dvukh potokov gaza raznoy skorosti i plotnosti. Izv. AN SSSR, OTN, No. 10, 1958.

3. G. N. Abramovich. Teoriya turbulentnykh struy. Fizmatgiz, 1960.

4. L. A. Vulis. K raschetu turbulentnykh svobodnykh struy szhimayemogo gaza. Izv. AN KazSSR, ser. energeticheskaya, No. 10, 1956.

5. Sh. A. Yershin. Eksperimental'no-teoreticheskoye issledovaniye turbulentnogo gazovogo fakela. Izv. AN KazSSR, ser. energeticheskaya, No. 1, 1960.

6. B. F. Glikman. Eksperimental'noye issledovaniye kondensatsii strui para v prostranstve, zapolnennom zhidkost'yu. Izv. AN SSSR, OTN, Energetika i Avtomatika, No. 1, 1959.

7. I. B. Palatnik. O rasprostranenii osesimmetrichnoy turbulentnoy strui konechnogo razmera v sputnom odnorodnom potoke. Izv. AN KazSSR, energeticheskaya, No. 2, 1960.

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